

A RECONCILIATION BETWEEN TWO COMPETING METHODS OF CPM CALCULATIONS

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ABSTRACT

This paper analyzes a basic difference between two methodological approaches that are in current use for CPM calculations, both in the professional practice and in the teaching CPM scheduling. Based on a literature review and on a preceding discussion of this issue it provides clarification to this inconsistency, illustrates the correct use of either method with an example, and provides guidance on how to avoid any confusion in CPM calculations by using the clarified definitions for periods and instances in time developed herein. This paper thus achieves a reconciliation of these two competing methods to benefit the entire scheduling community.

1 INTRODUCTION

Schedulers in their daily practice are determining a variety of time parameters for the activities that in their entirety make up a completed project. These parameters describe the ranges and instances in time when activities can occur, how flexible they are to changes, and how much of an impact delays in individual activities can have on the overall scheduled completion date of the project.

A recent discussion has shed light on the definition of these time parameters and how two significantly different methods of calculating them are in use. This paper makes a contribution to this discussion by reviewing both methods, clarifying their difference and how it potentially has been caused, and illustrating it with an example. It thus allows schedulers to be fully aware of the two methods and, regardless of which one is being used, achieve consistent and correct results.

2 LITERATURE REVIEW

The book by Feigenbaum (2002) on the use of scheduling principles with a common scheduling software was reviewed by Winter (2003c) and sparked a discussion (Winter 2003d) that succinctly pointed to a major perceived in-

consistency in critical path method (CPM) calculations, which can result in an unanticipated discrepancy of plus or minus one day in the computed results.

The fact that two such competing methods exist appears to have largely escaped the attention of the general scheduling community thus far (Winter 2003c). In this author's opinion, a factor that may have contributed to this confusion lies in the various labeling conventions used for time axes of schedule network diagrams. Typically a time axis is separated into units of equal length by the squares of the grid paper on which it is drawn by hand or by the hashmarks that are automatically added to diagrams in spreadsheet and scheduling software. Spreadsheet software, e.g. Microsoft® Excel, in fact allows switching between options where the hashmarks are plotted at the actual unit mark or between units (called "categories" in Excel). Three possibilities exist to indicate the date of a particular day when the hashmarks are plotted between units. The number of the day, i.e. its date, can either be printed above the hashmark at which the day starts on the time axis, in the middle between the two hashmarks (as common in the time axis layout used by scheduling software), or above the hashmark at which the day ends on the time axis. Only one reasonable possibility exists to indicate the date when the hashmark is plotted in the middle of the actual day on the time axis. It becomes obvious that precision in computational and labeling conventions is of the essence.

A review of standard books on scheduling, particularly focusing on its application for construction projects, reveals the currently predominating CPM calculation methodology. Feigenbaum (2002), whose book sparked the discussion that this paper addresses, presents bar charts in which the date is plotted in the middle between hashmarks that outline a cell along the time axis as used in spreadsheet software, which easily lends itself to drawing such bar charts. The activity-on-node (AON) diagrams and CPM calculations use the formula of $EF = ES + DUR$, i.e. the early finish date (EF) of an activity is equal to the early start date (ES) plus the duration (DUR) of said activity, is

used. In particular, the ES of the very first activity is shown as 0 in the AON diagrams, i.e. the schedule begins at the end of day 0 (which is equivalent to the start of day 1).

Callahan et al. (1992) in their diagrams use hashmarks that separate days at their beginning and ends. They place the dates over the hashmark at the end of a day. In the CPM calculations as provided in activity lists with CPM results they also use the formula of $EF = ES + DUR$. Halpin and Woodhead (1998) use this computational convention in their description and examples of CPM schedule calculations as well. While they do not provide schedule diagrams with hashmarks for days, they present a cash flow curve where the date is placed over the hashmark at the end of a month. The same use is found in Patrick (2004), where for CPM calculations the $EF = ES + DUR$ method is used consistently. All his diagrams carry an end-of-day date label. Buttewelwerth (2005) also uses the $EF = ES + DUR$ method for CPM calculations and provides screenshots of scheduling software as diagrams, in which the date is placed in the middle between hashmarks. Mubarak (2005) in general uses the convention in the manner described for the previous authors but deviates in his bar chart examples, where the number of the day is plotted in cells along the time axis.

This author notes that while in fact “there exists [sic] two different, formalized methods for computing a critical path method (CPM) schedule” (Winter 2003c, p24), the mathematical calculation is not at all dependent on the graphical display method used, whether being bar charts, activity-on-arrow (AOA) diagrams, or activity-on-node (AON) diagrams, the latter ones of which often also known as precedence diagramming method (PDM).

3 PROBLEM ANALYSIS

Differing from the publications examined in the previous section, the so-called AACE method (after the American Association of Cost Engineers) described by Winter (2003d) uses a mixed approach for counting and labeling dates, i.e. instances in time. As stated in a comparative table, “[a]ctivities begin at the start of the day and finish at the end of the work period” (Winter 2003d, p9). Underlying this definition is the notion that while the basic unit counted in scheduling calculations is the day, or more specifically weekdays and weekend (non-work) days within the chosen calendar, schedulers are aware that the actual work period is rather measured in hours during each day.

It is necessary to exactly describe the AACE method, which in this paper shall be called the mixed-day convention, to understand the approach and its difference in comparison with the method found in the literature as described in the previous section. Early start (ES) and late start (LS)

dates are defined as start-of-day instances in time. Early finish (EF) and late finish (LF) dates, however, are defined as end-of-day instances in time. In other words, an activity can have an ES of 1, a EF of 1, and a DUR of 1. For the formula introduced in the previous section to hold true (the duration is the difference between start and finish date), a correction of -1 has to be applied. Therefore, the mixed-day convention uses the formula $EF = ES + DUR - 1$ and its name is justified based on the mix of definitions for dates that it uses concurrently.

On the other hand, the convention found in the literature is an end-of-day convention, as derived from the fact that the examples give 0 as the ES for their very first activity. It would be equivalent using a pure start-of-day version instead, which would not change any of the CPM calculation formulas, but rather give values that are higher by 1 than the end-of-day convention for the ES, LS, EF, and LF dates. Both versions shall be called non-mixed convention.

As a clarification to the rule provided by Winter (2003d) that specifies that “[d]ay numbers for a CPM network begin with Day 0” this author suggests to speak of instances in time and that CPM scheduling calculations begin with the end of day 0. Days themselves are not instances in time but rather are a period in time.

Table 1: Scheduling Terminology Definitions

Item	Description
Activity	Individual element of the schedule
~ List	Enumeration of all activities of the schedule with at least their durations and successors
Duration	Time (usually in days) that activity takes to complete, depends on available resources
Date	Instance on time axis
Start ~	Early (ES) and late start (LS) possible
Finish ~	Early (EF) and late finish (LF) possible
Forward Pass	All activities occur as early as possible
Backward Pass	All activities occur as late as possible
Logic Path	Sequence and relationship between activities
	Single sequential chain of activities within the schedule
Critical ~	Longest sequential chain of activities within the schedule, which have zero total float
Float	Time (usually in days) that the start of an activity can flexibly be delayed within the schedule
Total ~	Possible delay of activity i without impacting project end, shared by all activities along one path, critical path has zero ~
Free ~	Possible delay of activity i without impacting any of its successors j under the Forward Pass

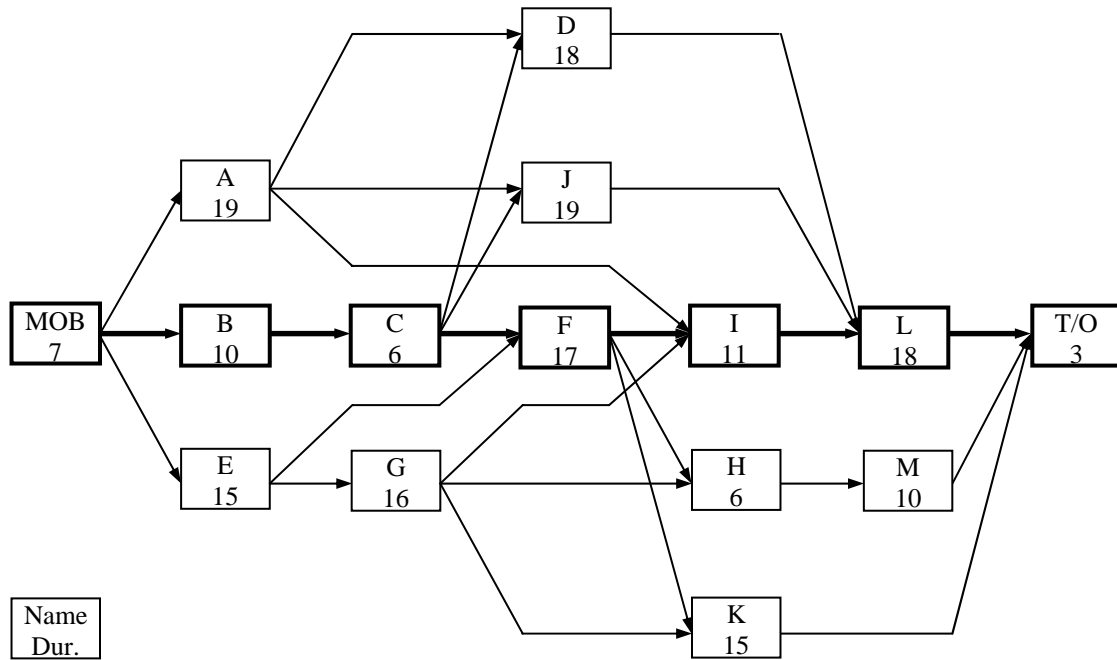


Figure 1: Sample Schedule Network

4 DEFINITIONS

The following list of definitions in Table 1 is suggested by this author as a clarification to the currently somewhat confusing terminology used in CPM scheduling calculations. The definitions for the various time parameters needed for CPM calculations are provided both verbally and, when appropriate, as a mathematical formula. The following equations give the formulas for the non-mixed convention (both in its end-of-day and start-of-day versions).

$EF = ES + DUR$ **Equation 1**

$LF = LS + DUR$ **Equation 2**

$TF_i = LS_i - ES_i = LF_i - EF_i$ **Equation 3**

$FF_i = (\text{minimum } ES_j) - EF_i$ **Equation 4**

For the mixed-day convention the formulas for EF, LF, and FF are modified slightly while the formula for TF remains unchanged.

$EF = ES + DUR - 1$ **Equation 5**

$LF = LS + DUR - 1$ **Equation 6**

$TF_i = LS_i - ES_i = LF_i - EF_i$ **Equation 7**

$FF_i = (\text{minimum } ES_j) - EF_i - 1$ **Equation 8**

where ES is the early start, EF is the early finish, LS is the late start, LF is the late finish, DUR is the duration, TF is the total float, FF is the free float, i is the index for an activity and j is the index for an activity directly succeeding activity i .

Table 2: Schedule Activity List with CPM Calculations based on End-of-Day Version of Non-Mixed Convention

Activity	Dur.	Successors	ES	LS	EF	LF	TF	FF
MOB.	7	A, B, E	0	0	7	7	0	0
A	19	D, I, J	7	13	26	32	6	0
B	10	C	7	7	17	17	0	0
C	6	D, F, J	17	17	23	23	0	0
D	18	L	26	33	44	51	7	7
E	15	F, G	7	8	22	23	1	0
F	17	H, I, K	23	23	40	40	0	0
G	16	H, I, K	22	24	38	40	2	2
H	6	M	40	53	46	59	13	0
I	11	L	40	40	51	51	0	0
J	19	L	26	32	45	51	6	6
K	15	T/O	40	54	55	69	14	14
L	18	T/O	51	51	69	69	0	0
M	10	T/O	46	59	56	69	13	13
T/O	3	N/A	69	69	72	72	0	0

Note: Boldface activities are on the critical path.

5 EXAMPLE

An example presented by Lucko (2005) in a discussion of the graphical layout of schedules will be developed further toward the correct and consistent application of the definitions of instances in time and periods of time in the previous section. Figure 1 gives the AON diagram for this simple 15-activity example. Table 2 gives the activity list, including mobilization and turnover activities and the results of the CPM calculation using the end-of-day convention, Table 3 gives the results of the CPM calculation using the start-of-day convention, and Table 4 gives the results of the CPM calculation using the mixed-day convention.

Table 3: Schedule Activity List with CPM Calculations based on Start-of-Day Version of Non-Mixed Convention

Activity	Dur.	Successors	ES	LS	EF	LF	TF	FF
Mob.	7	A, B, E	1	1	8	8	0	0
A	19	D, I, J	8	14	27	33	6	0
B	10	C	8	8	18	18	0	0
C	6	D, F, J	18	18	24	24	0	0
D	18	L	27	34	45	52	7	7
E	15	F, G	8	9	23	24	1	0
F	17	H, I, K	24	24	41	41	0	0
G	16	H, I, K	23	25	39	41	2	2
H	6	M	41	54	47	60	13	0
I	11	L	41	41	52	52	0	0
J	19	L	27	33	46	52	6	6
K	15	T/O	41	55	56	70	14	14
L	18	T/O	52	52	70	70	0	0
M	10	T/O	47	60	57	70	13	13
T/O	3	N/A	70	70	73	73	0	0

Note: Boldface activities are on the critical path.

Table 4: Schedule Activity List with CPM Calculations based on Mixed-Day Convention

Activity	Dur.	Successors	ES	LS	EF	LF	TF	FF
Mob.	7	A, B, E	1	1	7	7	0	0
A	19	D, I, J	8	14	26	32	6	0
B	10	C	8	8	17	17	0	0
C	6	D, F, J	18	18	23	23	0	0
D	18	L	27	34	44	51	7	7
E	15	F, G	8	9	22	23	1	0
F	17	H, I, K	24	24	40	40	0	0
G	16	H, I, K	23	25	38	40	2	2
H	6	M	41	54	46	59	13	0
I	11	L	41	41	51	51	0	0
J	19	L	27	33	45	51	6	6
K	15	T/O	41	55	55	69	14	14
L	18	T/O	52	52	69	69	0	0
M	10	T/O	47	60	56	69	13	13
T/O	3	N/A	70	70	72	72	0	0

Note: Boldface activities are on the critical path.

From reviewing Tables 2 through 4 it becomes abundantly clear that the two methods of using the end-of-day or start-of-day convention versus the mixed-day convention are equally valid and legitimate, since they only use a different counting convention for the same instances in time. Converting CPM results obtained from either convention back into actual calendar dates with a day-month-year format will give exactly the same values if done correctly. Particular care needs to be employed during this conversion process to also properly consider the difference between weekdays and weekend days, during which time passes but no work is performed. For example, for a calendar with a regular weekend consisting of Saturday and Sunday the end of Friday is equivalent to the start of Monday.

It needs to be stressed that what matters when using either one or the other convention is the consistency in doing so for the entire CPM calculation. Mixing the conventions within the same schedule will invariably lead to mathematical errors in the CPM calculation. Lucko (2005) has additionally pointed out a problem that arises from inconsistent use of the conventions for schedule diagrams. A non-existing overlap of directly succeeding activities was found in software-generated bar charts, which is hypothesized to have resulted from using the start-of-day convention in the CPM calculations but erroneously switching to the mixed-day convention when plotting the calculated dates into the bar chart with logic links without first converting the dates appropriately.

It is therefore strongly suggested to always refer to the non-mixed convention in its end-of-day or start-of-day versions and to the mixed-day convention, respectively, and to explicitly state these on top of each CPM schedule calculation and the graphical representation as bar chart, AOA diagram, or AON diagram based upon such calculation.

6 CONCLUSION

This paper has outlined two competing methods of performing CPM calculations, using either the end-of-day or start-of-day versions of the non-mixed naming convention to label the instances in time when activities begin and end, respectively, or the mixed-day convention that uses the start-of-day convention for ES and LS dates and the end-of-day convention for EF and LF dates. The importance of consistent use of either of these two methods has been stressed. An example has shown the numeric results of CPM calculations for both methods, including both versions of the non-mixed convention. Both methods are completely valid and will give correct results if used as prescribed. Attention is needed when transforming CPM calculation results back into calendar dates due to the distinction between weekdays and weekends that is introduced at this stage. Schedulers are free to choose either of the two methods to perform CPM calculations.

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